MATH 579: Combinatorics Exam 2

Please read the following instructions. For the following exam you may not use any papers, books, or computers. You may use a calculator. Please turn in **exactly four** problems. You must do problems 1-3, and one more chosen from 4-6. Number 7 is optional. Please write your answers on separate paper, make clear what work goes with which problem, adequately justify all answers, simplify all numerical answers as best you can, and put your name or initials on every page. You have 50 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 20 and 40. This will then be multiplied by $\frac{5}{2}$ for your exam score.

Turn in problems 1,2,3:

1. Find the number of solutions in integers to $x_1 + x_2 + x_3 = 10$, with each $x_i \geq 3i - 5$.
2. Calculate $B(5)$ using only its recurrence relation and $B(0) = 1$.
3. Prove that $S(n, n-2) = \frac{1}{24}n(n-1)(n-2)(3n-5)$, for all $n \geq 4$.

Turn in exactly one more problem of your choice:

4. Prove that the number of integer partitions of n into at most k parts, is equal to the number of integer partitions of n into any number of parts, each not larger than k.
5. Find all self-conjugate integer partitions of 23.
6. Determine the number of surjective functions $f : N \to K$, where $|N| = n, |K| = k$, the elements of N are indistinct, and the elements of K are distinct. Be sure to justify your answer.

You may also turn in the following (optional):

7. Describe your preferences for your next group assignment. (will be kept confidential)

Please keep this sheet for your records.